Third Semester B.E. Degree Examination, June 2012

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using Venn diagram, prove that for any set A, B, C, $A \Delta (B \Delta C) = (A \Delta B) \Delta C$. **(07 Marks)**

b. 75 children went to an amusement Park where they can ride on the merry go-round, roller-coaster and ferry wheel. It is known that 20 of them took all the rides and 55 have taken at least 2 of the 3 rides. Each ride cost Rs.0.5 and the total receipt of the Park was Rs.70. Determine the number of children who did not try any of these rides. **(07 Marks)**

c. A problem is given to 4 students A, B, C, D whose chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. Find the probability that (i) the problem is solved, (ii) problem is solved by only B and C. **(06 Marks)**

2 a. Prove the following logical equivalences without using truth table :

i) $[p \to (q \to r)] \Leftrightarrow [(p \wedge \sim r) \to \sim q]$

ii) $\sim [\sim \{(p \vee q) \wedge r\} \vee \sim q] \Leftrightarrow q \wedge r$ **(07 Marks)**

b. Test the validity of the following argument:

i) If there is strike by students, the exam will be postponed.
The exam was not postponed.
$\therefore$ There was no strike by students.

ii) Rita is baking a cake.
If Rita is baking a cake then she is not practicing her flute.
If Rita is not practicing her flute then her father will not buy her a car.
$\therefore$ Rita's father will not buy her a car. **(07 Marks)**

c. Define the following :

i) Modus pones
ii) Modus tollens
iii) NAND and NOR. **(06 Marks)**

3 a. Write down the following propositions in symbolic form and find its negation :

i) If all triangles are right angled, then no triangle is equiangular
ii) For all integer n, if n is not divisible by 2, then n is odd. **(07 Marks)**

b. Using quantifier method find whether following argument is valid :

If a triangle has two equal sides then it is isosceles.
If a triangle is isosceles then it has two equal angles.
The triangle ABC does not have two equal angles.
$\therefore$ ABC does not have two equal sides. **(07 Marks)**

c. Give a direct proof for each of the following :

i) For all integers K and l, if K, l are both even, then K + l is even.
ii) For all integers K and l, if K, l are even, then Kl is even. **(06 Marks)**

4 a. For all $n \in Z+$, show that if $n \geq 24$, then 'n' can be written as a sum of 5's and or 7's. **(07 Marks)**

b. Apply backtracking method to obtain an explicit formula for the sequence, defined by the recurrence relation $a_n = 7a_{n-1} + 1$, with initial condition $a_1 = 5$. **(07 Marks)**

c. For the Fibonacci sequence F_0, F_1, F_2 …….. Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(06 Marks)**

PART – B

5 a. Let R and S be binary relations on a set X. Then prove that :
i) $(R^{-1})^{-1} = R$ ii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ **(07 Marks)**

b. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1)\ R\ (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$
i) Verify that R is an equivalence relation on A × A.
ii) Determine the equivalence classes [(1, 1)], [(2, 2)], [(3, 3)]. **(07 Marks)**

c. Let S = {a, b, c} and A = P(S), define the relation R on A by ${}_xR_y$ if and only if $x \leq y$. Show that the relation is a partial order on A. Draw its Hasse diagram. **(06 Marks)**

6 a. Let A = {x / x is real and x ≥ -1} and B = {x / x is real and x ≥ 0}. Consider the function $f : A \to B$ defined by $f(a) = \sqrt{a+1}$, $\forall a \in A$. Show that f is invertible and determine f^{-1}. **(07 Marks)**

b. Define sterling number of 2nd kind, if |A| = 7, |B| = 4, find number of onto function from A to B. Hence find S(7, 4). **(07 Marks)**

c. Let f and g be functions from R to R (Reals) defined by f(x) = ax + b and $g(x) = 1 - x + x^2$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine values of a and b. Hence verify $(f \circ g) \circ h = f \circ (g \circ h)$ for positive values of a and b. **(06 Marks)**

7 a. If (G, ∗) is a group, prove that identity and inverse elements of a group are unique. **(07 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. Let $(G_1, \circ)$ and $(G_2, \circ)$ be two groups with respective identities e_1, e_2 and if $f : G_1 \to G_2$ is homomorphism then prove that (i) $f(e_1) = e_2$, (ii) $f[a^{-1}] = [f(a)]^{-1}$, $\forall a \in G$. **(06 Marks)**

8 a. The generator matrix for an encoding function $E : z_2^3 \to z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

i) Find the code words assigned to 110 and 010.
ii) Obtain the associated parity check matrix. **(07 Marks)**

b. Prove that the set z with binary operations ⊕ and ⊙ defined by x ⊕ y = x + y – 1, x ⊙ y = x + y – xy is a commutative ring with unity. **(07 Marks)**

c. Show that i) z_6 is not a field and ii) z_5 is an integral domain under the multiplication (•). **(06 Marks)**

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